Experimental verification of predicted oscillations near a spin resonance

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The Chao matrix formalism allows analytic calculations of a beam’s polarization behavior inside a spin resonance. We recently tested its prediction of polarization oscillations occurring in a stored beam of polarized particles near a spin resonance. Using a 1.85 GeV/c polarized deuteron beam stored in COSY, we swept a new rf solenoid’s frequency rather rapidly through 400 Hz during 100 ms, while varying the distance between the sweep’s end frequency and the central frequency of an rf-induced spin resonance. Our measurements of the deuteron’s polarization near and inside the resonance agree with the Chao formalism’s predicted oscillations.

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Polarized stored hadron and lepton beams often provide the best technique for studying the spin dependence of some hadronic interactions in the 1 GeV/c to 1 TeV/c region. Polarized beam experiments at storage rings [1–5] need the ability to precisely control the beam’s polarization. A stored beam’s polarization can be manipulated in a well-controlled way by ramping an rf magnet’s frequency through an rf-induced spin resonance.

The Froissart-Stora (F-S) formula [6] has been widely used to calculate a beam’s polarization after crossing a spin resonance. However, it is only valid for a constant-rate linear crossing from far below to far above the resonance. Chao’s new matrix formalism was proposed [7] to deal with conditions where the F-S formula is not valid; the Chao formalism can be used to calculate the spin dynamics anywhere inside a piece-wise linear resonance crossing. An earlier experiment [8] at COSY first tested the Chao formalism’s predicted spin behavior by sweeping an rf dipole’s frequency near or through an rf-induced spin resonance. That experiment suggested that a faster crossing rate was needed to test its striking prediction of large polarization oscillations near the resonance. This paper describes a faster crossing rate experiment at COSY.

In an ideal flat circular storage ring or accelerator, with no horizontal magnetic fields, each particle’s spin precesses around the vertical magnetic fields of the ring’s bending dipoles. The spin tune νs, which is the number of spin precessions during one turn around the ring, is proportional to the particle’s energy: νs = Gγ, where G = (g − 2)/2 is its gyromagnetic anomaly and γ is its Lorentz energy factor. Horizontal rf magnetic fields can induce an rf spin resonance [6, 9, 10], which can be used to spin-manipulate the stored particles [11–16]. For deuterons, the rf-induced spin resonance’s frequency is

\[ f_r = f_0 (k \pm G_d \gamma) \] (1)

where \( f_0 \) is the deuteron’s circulation frequency, \( k \) is an integer, and \( G_d = -0.142987 \).

Ramping an rf magnet’s frequency through a spin resonance with resonance strength \( \varepsilon \), can rotate a stored beam’s polarization. When its frequency is ramped at a constant rate, during a ramp time \( \Delta t \), by a range \( \Delta f \), from far below to far above a resonance, the Froissart-Stora equation [6] can relate the beam’s initial vector polarization \( P_i \) and its polarization \( P_f \) after crossing the resonance,

\[ P_f = P_i \left\{ 2 \exp \left[ -\frac{(\varepsilon f_r)^2}{\Delta f / \Delta t} \right] - 1 \right\} . \] (2)

As discussed earlier [8], Chao’s matrix formalism [7] for spin dynamics allows one to analytically solve the spin equation of motion near an isolated spin resonance, if its crossing can be expressed as a series of linear segments. Each segment must have a fixed or linearly changing distance between the spin tune \( \nu_s = G \gamma \) and the resonance.
sequent to obtain the final polarization for different distances between the ramp’s end-frequency $f_{\text{end}}$ and the resonance’s center $f_r$.

We recently tested Chao’s matrix formalism with the technique shown in Fig. 1 using our new rf solenoid; it is a 25-turn air-core water-cooled copper coil, of length 57.5 cm and average diameter 21 cm. Its inductance was $41 \pm 3 \mu$H; and its longitudinal rf magnetic field was about 1.25 mT at its center. It was part of an RLC resonant circuit, which operated near 917 kHz, typically at an rf voltage of 5.7 kV rms producing a longitudinal rf $f B_{\text{rms}} d l$ of 0.69 $\pm$ 0.05 T·mm.

The other apparatus for this experiment, including the COSY storage ring [17–20], the EDDA polarimeter [21, 22], the electron cooler [23], the low energy polarimeter [24], the injector cyclotron, and the polarized ion source [25–27] were shown in Fig. 4 of ref [8]. The beam from the polarized $D^-$ ion source was accelerated by the cyclotron to 75.7 MeV and then strip-injected into COSY. The Low Energy Polarimeter measured the $D^-$ beam’s polarization before injection into COSY to monitor the cyclotron’s and ion source’s stability.

The EDDA polarimeter [21, 22] measured the beam’s polarization in COSY. We reduced its systematic errors by repeatedly cycling the beam produced by the polarized deuteron ion source through five different vector $P_V$ and tensor $P_T$ vertical polarization states:

$$ (P_V, P_T) = (0, 0), (+1, +1), (-\frac{1}{2}, -1), (-\frac{3}{2}, 0), (-1, +1). $$

The asymmetry measured in the $(0, 0)$ spin state was subtracted from the other measured asymmetries, in each 20 ms time-bin, to correct for detector efficiencies and beam motion asymmetries in the EDDA polarimeter.

In COSY, the deuterons’ average circulation frequency $f_c$ was 1.14743 MHz at 1.85 GeV/c, where their Lorentz energy factor was $\gamma = 1.4046$. For these parameters, the spin tune $\nu_s = G d \gamma$ was $-0.20084$. Thus, Eq. (1) implies that the $k = 1$ spin resonance’s central frequency should be very near $f_r = (1 + G d \gamma) f_c = 917.0$ kHz.

We measured the rf solenoid’s strength $\mathcal{E}$ by measuring the polarization after ramping its frequency through the resonance with various ramp times $\Delta t$ with its frequency range $\Delta f$ and voltage fixed, as shown in Fig. 2. We then fit these data to Eq. (2), the Froissart-Stora equation [6] to obtain the measured value of $\mathcal{E}$.

To study the Chao formalism’s predicted dependence on the beam’s momentum spread $\Delta p/p$, we varied the 20.6 keV electron Cooler’s on-time at injection. It cooled the deuterons’ emittances both longitudinally and transversely for 15 or 25 s. The deuterons were then accelerated to 1.85 GeV/c. The rf acceleration cavity was off and shorted during COSY’s flat-top; thus, there were no synchrotron oscillations.

We tested the Chao formalism by ramping the rf solenoid’s frequency over a range $\Delta f$, which started at $f_{\text{start}}$ (well outside the rf spin resonance centered at $f_r$) and ended at $f_{\text{end}}$ near or inside the resonance, as shown in Fig. 1. For each $f_{\text{end}}$ data point, both $\Delta f$ and the ramp time $\Delta t$ were held fixed, at 400 Hz and 100 ms, respectively, while $f_{\text{end}}$ was set to one of the values shown in Figs. 3 and 4. After $f_{\text{rf}}$ reached $f_{\text{end}}$, the rf solenoid was turned off abruptly (in a few $\mu$s) to preserve the vertical polarization at that instant. We then measured the deuterons’ vector asymmetry in all five $(P_V, P_T)$ states. The resulting final vector polarization $P_f$ for each nonzero spin state is plotted vs $f_{\text{end}}$ in Figs. 3 and 4.

We first calculated the Chao formalism’s prediction for $\Delta p/p = 0$ by inserting, into Eqs. (4)-(9) of ref [8], our measured $\mathcal{E} = 1.06 \times 10^{-3}$ from Fig. 2, our $\Delta f$ of 400 Hz, and our $\Delta t$ of 100 ms. To take into account the beam’s momentum spread $\Delta p/p$, we next folded this result to-
FIG. 3: Measured 1.85 GeV/c deuteron vector polarizations plotted vs rf-solenoid end frequency \( f_{\text{end}} \). Its ramp time \( \Delta t \) was 100 ms; its frequency range \( \Delta f \) was 400 Hz, and its \( \varepsilon \) was \( 1.06 \times 10^{-5} \). The Chao formalism fits, shown by lines, gave a resonance frequency \( f_r \) of 916 986.6 ± 0.5 Hz and a Gaussian \( \delta f_{\Delta p} \) of 32 ± 2 Hz fwhm. Only the data’s statistical errors were used to calculate \( \chi^2/(N-2) \). E-cooling was on for 15 s.

FIG. 4: Measured 1.85 GeV/c deuteron vector polarizations again plotted vs \( f_{\text{end}} \) as in Fig. 3. With 25 s e-cooling, the fits, shown by lines, gave \( f_r \) of 916 985.3 ± 0.5 Hz and \( \delta f_{\Delta p} \) of 23 ± 2 Hz fwhm. The solid line shows the prediction \([7, 8]\). The large difference clearly demonstrates the Chao formalism’s sensitivity to changes in \( \varepsilon \).

For each cooling-time, we also measured the final polarization \( P_f \) after the rf solenoid was run at many different fixed frequencies to independently determine the resonance’s center \( f_r \) and its fwhm-width \( \Delta f \). These data are plotted vs the rf frequency \( f_r \) in Fig. 6, along with their fit values of \( f_r \) and \( \Delta f \) for the 15 and 25 s cooling-times. The fit widths of 41 ± 1 and 29 ± 1 Hz for the 15 s cooling-time, respectively, were equal to the folding together of the resonance’s natural width \( 2\varepsilon f_r = 24 \) Hz and the \( f_r \)-spreads, \( \delta f_{\Delta p} \), obtained by fitting the data in Figs. 3 and 4. Adding this 24 Hz width in quadrature with the 32 and 23 Hz \( \delta f_{\Delta p} \) values gives 40 ± 2 and 33 ± 2 Hz; these seem consistent with the 41 ± 1 and 29 ± 1 Hz from Fig. 6, for cooling-times of 15 and 25 s, respectively.
For 25 s cooling, the $f_r$ of 916 985.3 ± 0.3 Hz from Fig. 4 and the $f_r$ of 916 990 ± 10 Hz from Fig. 6b differ by only 5 Hz. (The Fig. 6b and Fig. 4 data were obtained sequentially.) But, for 15 s cooling, with 102 hours between the Fig. 3 and Fig. 6a data runs, the $f_r$ values are 916 986.6 ± 0.3 Hz and 917 010 ± 10 Hz, respectively. This 23 Hz shift may be due to COSY’s long-term stability.

In summary, we tested the Chao formalism’s predictions for the polarization’s behavior while crossing an isolated spin resonance, where the Froissart-Stora formula is not valid. Using 1.85 GeV/c vertically polarized deuterons stored in COSY, we ramped an rf solenoid’s frequency through a range $\Delta f$ ending near a spin resonance; the magnitudes of both $\Delta f$ and the ramp time $\Delta t$ were fixed, while we varied the ramp’s distance from the resonance. In Figs. 3-5 we found large oscillations in the polarization in good agreement with the predictions of the Chao formalism [7, 8].

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