Dear colleagues,

I think I know the answer to the $G\gamma$ vs. $(1 + G\gamma)$ question.

There is no contradiction between Ernest's $(1 + G\gamma)$ result for the factor describing dependence of spin resonance strength on the radial B-field, and the $G\gamma$ result of Kondratenko and others for that very dependence. Simply, Ernest's result is intermediate, not final.

According to Eq. (14) of Ernest's e-mail of 7/25/07, the main resonance term arising from the radial field is

$$S_2(1 + G\gamma)z'' / R.$$  

So we will concentrate on this term.

Consider the case $G=0$. Let a dipole with an oscillating $B_R$-field be located at some azimuth, and let only vertical beam oscillations be involved in our spin resonance operations. Then the spin-resonance frequency of the $B_R$-field equals $n\omega_c$, where $n$ is an integer, and $\omega_c$ is the revolution frequency,

$$B_R(t) = B_{R0} \cos n\omega_c t.$$  

Due to synchrotron bunching, the particles perceive this perturbation (located at one point) as if it is constant in time. And due to vertical focusing, every particle shifts its vertical positions along the equilibrium orbit in such a way that the average radial $b_R$-field, met by this particle at that shifted orbit, equals zero:

$$\langle b_R(t) \rangle = 0.$$  

(This follows from the vertical stability condition, $< dp_z / dt > = 0$. We do not take into account synchrotron oscillations of the velocity in the $v \times B$.) Here, $b_R(t)$ contains both original perturbation, $B_R(t)$, and the perturbation induced by that $B_R(t)$ due to the shifts of the vertical closed orbit inside the gradient lenses and magnets with gradients. The induced field is therefore distributed along the ring.
Now, the physical picture is the following. The angle between spin and velocity is constant, (see, e.g., (11.171) in Jackson). The planar projection of this angle is constant in time, if we neglect free radial betatron oscillations. Therefore, the Fourier spectrum of the (g-2) rotations (that is, rotations in the horizontal plane) contains only zero frequency. But according to (15), the radial $b_r$-field does not contain zero frequency. Hence, there is no spin resonance.

In terms of Ernest 's Eq. (14), in the case of $G=0$, there is no resonance between $S_2$ and $z''/R$ (which is obvious when $R=constant$, since $z''$ never has a component constant in time and $S_2 = constant$ when $G=0$.)

Thus, Eq. (14) is correct, but this is not the end of story. If we accurately calculate $z''/R$ and $S_2(t)$—by solving all three spin eq.'s together—we will undoubtedly arrive at a final formula such that the spin resonance strength goes down to zero when $G --> 0$. This means that at least at the small $G\gamma$'s, the resonance strength is proportional to $G\gamma$, not $(1 + G\gamma)$.

We need to consider all three spin equations—not only to get the resonance shift of the (g-2) tune, but to check whether there are any effects of the noncommutativity of rotations. Such effects are readily visible in the matrix technique of calculation, but may be "hidden" in the continuous-equations technique.

All the best to all,
Yuri
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