Spin resonance strength due to an rf solenoid with stored 1.85 GeV/c deuterons

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Recent studies reported huge disagreements between the spin resonance strengths $\varepsilon_{FS}$ measured in controlled Froissart-Stora frequency sweeps and the theoretical values $\varepsilon_{Bdl}$ calculated from the rf magnet’s $\int Bdl$ using the previously-accepted formulae. The measured ratios of $\varepsilon_{FS}/\varepsilon_{Bdl}$ ranging from 0.15 to 170 found in these studies resulted in careful re-examinations of these formulae. The experiments at COSY were a systematic study of spin resonance strengths. This paper presents the results from Froissart-Stora sweeps with stored 1.85 GeV/c polarized deuterons using a new rf-solenoid magnet; the measured ratio $\varepsilon_{FS}/\varepsilon_{Bdl}$ was 1.02 ± 0.05 over a wide range of accelerator settings. Similar sweeps using an rf-dipole magnet gave measured values of $\varepsilon_{FS}/\varepsilon_{Bdl}$ of 0.15 ± 0.01. These rf-solenoid measurements firmly support the “BNL” factor of 2 in the formulae for $\varepsilon_{Bdl}$ and validate the experimental calibration of the earlier rf-dipole results at COSY.

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Polarized stored hadron and lepton beams often provide the best technique for studying the spin dependence of hadronic interactions in the 1 GeV/c to 1 TeV/c region. Polarized beam experiments at storage rings [1–5] need the ability to precisely control the beam’s polarization. A stored beam’s polarization can be manipulated in a well-controlled way by ramping an rf magnet’s frequency through an rf-induced spin resonance. To design experiments utilizing this technique, one needs to be able to accurately predict the strength of such a resonance from the rf magnet’s magnetic field integral $\int Bdl$.

Recent systematic studies [6, 7] of rf-magnet-induced spin resonance strengths with protons, deuterons and electrons reported huge disagreements between the spin resonance strengths $\varepsilon_{FS}$ measured in controlled Froissart-Stora frequency sweeps and the theoretical values $\varepsilon_{Bdl}$ calculated from each rf magnet’s magnetic field integral $\int Bdl$ using the previously-accepted formulae [8–11]. For protons with an rf dipole, the measured $\varepsilon_{FS}/\varepsilon_{Bdl}$ ratio was always greater than 1 and was greatly enhanced [6] near a 1st-order intrinsic spin resonance. Similar studies with deuterons and an rf dipole showed that the $\varepsilon_{FS}/\varepsilon_{Bdl}$ ratio was also enhanced very near an intrinsic spin resonance [7]. However, far from any intrinsic spin resonances, the deuteron $\varepsilon_{FS}/\varepsilon_{Bdl}$ ratio had a value of 0.15 ± 0.01 [7], thus, disagreeing with the prediction by a factor of about 7. These observed disagreements involving $\varepsilon_{FS}/\varepsilon_{Bdl}$ ratios ranging from 0.15 to 170 resulted in careful theoretical re-examinations of the $\varepsilon_{Bdl}$ formulae. To provide experimental guidance in the search for correct versions of the $\varepsilon_{Bdl}$ equations, our recent experiment made Froissart-Stora sweeps with stored 1.85 GeV/c polarized deuterons using a new rf-solenoid magnet.

In any flat storage ring or circular accelerator with no horizontal magnetic fields, each beam particle’s spin precesses around the vertical fields of the ring’s dipole magnets. The spin tune $\nu_s$, which is the number of spin precessions during one turn around the ring, is proportional to the particle’s energy

$$\nu_s = G\gamma ,$$

(1)

where $G = (g - 2)/2$ is the particle’s gyromagnetic anomaly ($G_p = 1.792847, G_e = 1.15967 \times 10^{-3}$ and $G_d = -0.142987$) and $\gamma$ is its Lorentz energy factor.
The vertical polarization can be perturbed by an rf magnet’s horizontal rf field. This perturbation can induce an rf spin resonance [12–14], which can be used to spin-manipulate the stored particles [6, 7, 15–17]; the resonance’s frequency is

\[ f_r = f_c (k \pm \nu_s), \tag{2} \]

where \( f_c \) is the particle’s circulation frequency and \( k \) is an integer.

Ramping an rf magnet’s frequency from far below to far above \( f_r \) can rotate a stored beam’s polarization. The Freissart-Stora (FS) equation [12] relates the beam’s initial polarization \( P_i \) to its final polarization \( P_f \) after crossing the resonance,

\[ P_f = P_i \left\{ 2 \exp \left[ \frac{(\pi \varepsilon_{FS} f_c)^2}{\Delta f / \Delta t} \right] - 1 \right\}, \tag{3} \]

where \( \Delta f \) is the ramp’s frequency range during the ramp time \( \Delta t \) and \( \varepsilon_{FS} \) is the resonance strength.

For an ideal flat circular accelerator, with no horizontal \( B \)-fields, the resonance strength \( \varepsilon_{Bdd} \) due to a short rf solenoid’s longitudinal \( B \)-field or a short rf dipole’s transverse \( B \)-field was thought to be given by [8–11]

\[
\text{Solenoid: } \varepsilon_{Bdd} = \frac{1}{\pi \sqrt{2}} \frac{e(1 + G)}{p} \int B_{rms} dl, \tag{4}
\]

\[
\text{Dipole: } \varepsilon_{Bdd} = \frac{1}{\pi \sqrt{2}} \frac{e(1 + G\gamma)}{p} \int B_{rms} dl, \tag{5}
\]

where \( e \) is the particle’s charge, \( p \) is its momentum, and \( \int B_{rms} dl \) is the rf magnet’s rms magnetic field integral in its rest frame. Equations (4) and (5) were widely believed to be correct for a planar storage ring with negligible radial or longitudinal magnetic fields, except for the rf magnet. This seems to be a good assumption for an rf solenoid but it may not be valid for an rf dipole, which, in addition to its direct effect on the spin, excites coherent vertical beam oscillations. These oscillations may lead to an additional effect on the spin from other ring elements.

Note that there was a theoretical disagreement [8–11, 18] about the corrector factor of 2 in both Eqs. (4) and (5). Some authors [19, 20] derived similar equations, but with no factor of 2 in the denominator.

The apparatus, used for this experiment, included the COSY storage ring [21–24], the EDDA detector [25, 26], the electron Cooler [27], the Low Energy Polarimeter, the injector cyclotron, the polarized ion source [28–30], and the new rf solenoid. The beam emerging from the polarized D$^- \) ion source was accelerated by the cyclotron to COSY’s injection energy of about 75.7 MeV and then strip-injected into COSY. The Low Energy Polarimeter measured the beam’s polarization before the injection; this monitored the stable operation and polarization of the ion source and cyclotron. We sometimes used COSY’s electron cooler, which reduced the beam’s size and momentum spread at injection energy. A 20.6 keV electron beam cooled the deuteron beam’s transverse emittance and reduced its longitudinal momentum spread by more than a factor of 27.

We manipulated the deuteron’s polarization using our new rf solenoid, which was a 25-turn air-core water-cooled copper coil, of length 57.5 cm and average diameter 21 cm. Its inductance was 41 ± 2 Hz. It was part of an RLC resonant circuit, which operated near 917 kHz, typically at an rf voltage of 5.7 kV rms producing an rf longitudinal magnetic field of about 1.17 mT rms at its center and an rf \( \int B_{rms} dl \) of 0.67 ± 0.057 T·mm.

The EDDA polarimeter [25, 26] measured the beam’s polarization in COSY. We reduced its systematic errors by repeatedly cycling the beam produced by the polarized deuteron ion source through five different vector \( P_V \) and tensor \( P_T \) vertical polarization states:

\[
(P_V, P_T) = (0, 0), (+1, +1), (-1, -1), (-\frac{1}{2}, 0), (-1, +1). \]

The asymmetry measured in the (0, 0) spin state was subtracted from the other measured asymmetries, in each 20 ms time-bin, to correct for detector efficiencies and beam motion asymmetries in the EDDA polarimeter.

In COSY, the deuteron’s average circulation frequency \( f_c \) was 1.14743 MHz at 1.850 GeV/c, where their Lorentz energy factor was \( \gamma = 1.4046 \). For these parameters, the spin tune \( \nu_s = G\gamma = -0.20084 \). Thus, Eq. (2) gives that the \( k = 1 \) spin resonance’s central frequency should be \( f_r = (1 + G\gamma) f_c \) near 917.0 kHz.

We determined the resonance’s position by measuring

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**FIG. 1**: Measured vector deuteron polarizations at 1.85 GeV/c plotted vs rf-solenoid frequency \( f_r \). (a) Fits to a 5th-order Lorentzian give \( f_r \) of 916.988 ± 10 Hz and \( w \) of 86 ± 2 Hz FWHM. (b) Fits to a 2nd-order Lorentzian give \( f_r \) of 917.010 ± 10 Hz and \( w \) of 41 ± 1 Hz FWHM. The data points’ errors are purely statistical; the errors in \( f_r \) and \( w \) include estimates of systematic errors due to COSY’s stability at the few Hz level.
FIG. 2: Measured vector deuteron polarizations at 1.85 GeV/c plotted vs rf solenoid’s ∆f for 4 different spin states with electron cooling off. The rf solenoid’s ∆f was 200 Hz; its ∫Bdl was 0.67 ± 0.05 T·mm; thus, Eq. (4) gives ε_{Bdl} of (1.04 ± 0.07) × 10^{-5}. The fit to Eq. (3) gives ε_{FS} of (1.07 ± 0.01) × 10^{-5}.

The polarization with the rf solenoid set at different fixed frequencies. These data are plotted vs the rf frequency f_r in Fig. 1 for the electron cooling off and on. Figure 1 also shows the data fit values of the resonance’s center f, and its FWHM-width w for both e-cooling conditions. Note that f_r shifted by about 22 Hz between the e-cooling off and on studies; this shift is probably due to COSY’s long-term stability. As shown in Fig. 1, the e-cooling reduced the resonance’s w from 86 ± 2 to 41 ± 1 Hz FWHM.

We obtained the resonance strength ε_{FS} by first measuring the final beam polarization P_f after ramping the rf solenoid’s frequency by a range ∆f during a time ∆t through the spin resonance. The measured dependence of P_f on ∆t was then fit to Eq. (3) with ε_{FS} as a fit parameter. An example of such a curve is shown in Fig. 2. Fitting the data in Fig. 2 to Eq. (3) gives ε_{FS} of (1.07 ± 0.01) × 10^{-5} while using the rf solenoid’s ∫Bdl in Eq. (4) gives ε_{Bdl} of (1.04 ± 0.07) × 10^{-5}. Note that the measured and predicted ε values agree within error; their ratio ε_{FS}/ε_{Bdl} is 1.03 ± 0.07.

The earlier rf-dipole ε_{FS}/ε_{Bdl} data [7] for the different values of the frequency sweep range ∆f are shown in Fig. 3. All these rf-dipole data were anomalously low by a factor of about 6.7 independent of ∆f or the beam’s momentum spread ∆p/p. The fit to all rf-dipole points gave a resonance strength ratio of 0.15 ± 0.01. However, the single rf-solenoid point from the Indiana University Cyclotron Facility (IUCF) was near to 1. Thus, to check the IUCF point and to test whether an rf solenoid has an anomalous strength value with the deuterons, we used COSY’s new rf solenoid to measure the ε_{FS}/ε_{Bdl} ratio for different values of ∆f with the electron cooling both on and off. These data are shown in Fig. 3 along with the rf-dipole data. The fit to all COSY rf-solenoid points gave an ε_{FS}/ε_{Bdl} ratio of 1.02 ± 0.05. As with the rf dipole, the rf-solenoid data were independent of both ∆f and ∆p/p.

We next measured ε_{FS}, as in Fig. 2, for different values
of the vertical betatron tune $\nu_y$; $\varepsilon_{Bdl}$ was again obtained using each data point's $\int B dl$ in Eq. (4). The $\varepsilon_{FS}/\varepsilon_{Bdl}$ ratios are plotted against $\nu_y$ in Fig. 4 along with the earlier rf-dipole data [7]. Notice the nearby $\nu_y = \nu_y - 4$ first-order intrinsic spine resonance for deuterons. The rf-dipole strength is enhanced near the $\nu_y = \nu_y - 4$ resonance. However, far from it, the rf-dipole strength is again anomalously low. Aside for a single point almost exactly on top of the $\nu_y = \nu_y - 4$ resonance, the rf-solenoid strength has no dependence on $\nu_y$. This is expected because, unlike an rf dipole, an rf solenoid does not excite coherent vertical beam oscillations, which would cause the strength enhancement. The fit to all rf-solenoid points, excluding the point on the resonance, gives an $\varepsilon_{FS}/\varepsilon_{Bdl}$ ratio of $1.02 \pm 0.05$.

In summary, our recent experiment made Froissart-Stora sweeps with stored 1.85 GeV/c polarized deuterons using a new rf-solenoid magnet. The measured $\varepsilon_{FS}/\varepsilon_{Bdl}$ ratio was $1.02 \pm 0.05$ over a wide range of accelerator settings. However, similar sweeps using an rf-dipole magnet earlier gave measured $\varepsilon_{FS}/\varepsilon_{Bdl}$ values of $0.15 \pm 0.01$ [7]. As shown in Figs. 3 and 4, these new rf-solenoid data's precise agreement with Eq. (4) firmly supports that Eq. (4) is correct with the 'BNL' factor of 2 [8–11, 32]. Moreover, these rf-solenoid measurements validate the experimental calibration of the earlier rf-dipole results [6, 7], which disagree with Eq. (5) by a factor of about 7. Thus, one must now conclude that Eq. (5) is incorrect. This problem was discussed by several authors [11, 33–35] who stated that Eq. (5) is not correct because it ignores the enhanced strength due to the rf dipole, which increases the coherent vertical betatron oscillations; these drive the particles further into the radial magnetic fields of the ring quadrupoles. Recent calculations [31] and simulations [36] are now trying to correctly take into account the vertical betatron oscillations driven by the rf dipole.

The rf-dipole $\varepsilon_{FS}/\varepsilon_{Bdl}$ data for deuterons [6, 7] also pointed to another problem with Eq. (5). It now appears that its widely accepted $(1 + G_\gamma)$ factor is not correct. This was first pointed out by A.M. Kondratenko who noted that, with no magnet end fields, the "leading term" in $\varepsilon_{Bdl}$ is proportional to $G_\gamma$; then $\varepsilon_{Bdl}$ goes to zero when $G$ goes to zero. Other papers [9–11] said that $\varepsilon_{Bdl}$ was proportional to $(1 + G_\gamma)$; then it does not go to zero. Although, $G$ is certainly not zero for protons, deuterons or electrons, one was concerned by this apparent paradox. The paradox now seems to be resolved theoretically [31, 37–40]; it now appears that Eq. (5) must be replaced by a much more complex calculation that properly includes the enhancement of $\varepsilon_{Bdl}$ due to the rf dipole, which causes coherent vertical forced oscillations of the beam, and the effect of all radial and longitudinal magnetic fields in the rest of the ring. Figure 4 shows one such calculation by A.M. Kondratenko [31], using a simplified COSY lattice. The term 1 in $(1 + G_\gamma)$ is mostly canceled by part of the contributions from the vertical oscillations in the ring quadrupoles. This cancelation is not obvious; thus, its resolution was somewhat subtle. In the future, we hope to provide further experimental guidance and then confirmation for a correct and hopefully analytic version of Eq. (5); we also hope to confirm Eq. (4) for protons with an rf solenoid.

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